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TITLE: RECENT STUDIES OF THE GAMMA-RAY LASER PROBLEM

[illegible]

June

MASTER

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Addendum to LA-UR-80-96

"Recent Studies of the Gamma-Ray Laser Problem"

By

**George C. Baldwin, Laurie E. McNeil, Johndale C. Solem,
and Bergen R. Suydam**

FIGURES

(Used as Vu-graf transparencies)

TEXT OF TALK

(Accompanying figures indicated)

**Note: This paper is to be presented at the XI International Quantum
Electronics Conference as Paper #DD-10, June 23, 1980.**

FIGURE 1

The possibility of extending the laser principle into the hard x-ray region above a few keV depends upon the ability of a pump to create the critical density of population inversion for which gain overcomes loss by absorption. Although this critical density decreases with the wavelength of the radiation to be stimulated, the power required to generate it depends upon the lifetime of the state being pumped. The lifetimes of inner-shell vacancies of atoms are very short.

Nuclear states, on the other hand, have much longer lifetimes, ranging from fractions of picoseconds to millenia.

FIGURE 2

Moreover, in the so-called "recoilless" or "Mössbauer" transitions of nuclear isomers, it is observed that the resonance cross section often exceeds the nonresonant absorption cross section by several orders of magnitude: just the condition for lasing in an inverted population. If, other things being equal, the absorber foil of a Mössbauer experiment contained an excess of excited states, then, instead of the absorption dip normally observed at resonance, there would be an increase of intensity; amplification by stimulated emission would be achieved.

The problem in making a gamma-ray laser is therefore simply that of obtaining an inverted population without inhibiting the Mössbauer effect.

FIGURE 3

In 1973, several proposals for accomplishing this appeared in Soviet sources; two proposed to pump Mössbauer transitions in situ by means of neutron bursts. All known Mössbauer transitions have lifetimes shorter than ten microseconds; longer-lived ones have unduly broadened lines. For transitions having energy of the order of 10 keV or less, gamma emission is almost entirely recoilless, provided the temperature of the solid host can be kept below the Debye temperature.

Pumping must therefore create inversion without undue temperature rise; otherwise the stimulation cross section will fall below that for nonresonant absorption. Gol'danskii and Kagan proposed to employ a rigid solid host of low gamma-ray absorption, to furnish heat capacity, in the form of a thin filament, to define a beam and allow escape of radiation generated by neutron capture.

FIGURE 4

Their estimate of the required number of neutron captures, and of that part of the energy released upon their capture (including 8 MeV binding energy and recoil from neutron impact) that could not escape from a thin filament, led to the conclusion that the temperature rise could be tolerated, if the neutron energy could be kept low, so as to avoid heating by recoil.

FIGURE 5

They estimated the neutron fluence to pump the Mössbauer transition in Ta-181; this required knowledge of the neutron capture cross section of its parent, Ta-180. The result was $3(19)$ neutrons cm^{-2} , below 100 eV, to be delivered within its 10-microsecond mean lifetime. This, they recognized, would require a nuclear explosion for the neutron source.

FIGURE 6

Although they felt that direct pumping might nevertheless be feasible, Gol'danskii, Kagan, and Namiot also showed how one might greatly reduce the neutron fluence requirement. They proposed a two-stage pumping process, in which neutron capture generates the Mössbauer isotope in a separate region, where its radiation interacts only by nonresonant absorption before entering the graser. There, Mössbauer absorption, with a much higher cross section, produces the excitation in a thin region; the resultant density of excited states is much higher than the density of captures in the converter.

FIGURE 7

Several other proposals have appeared that elaborate on the two basic proposals of Gol'danskii and Kagan; in all, neutrons, produced in fission or fusion reactions, must first be moderated and then captured to form the excited population.

FIGURE 8

At LASL, we have studied mainly the first two neutron-burst-pumped proposals, although we have also given consideration to alternatives that would employ long-lived transitions. Those would first require developing methods for reducing the linewidth of the Mössbauer radiation. Today, I shall discuss only the neutron-burst-pumped proposals, they involve aspects quite different from conventional lasers.

FIGURE 9

First: Even if an extremely brief burst of neutrons could be generated, the neutrons must first be slowed down in order for the capture cross section to be appreciable and to avoid overheating the graser. Fluctuations of moderating times make the pumping sustained, rather than pulsed. No previous work has considered this aspect.

Second: The neutrons are captured into a highly excited state; the resulting cascade of capture gamma radiation can populate either or both levels. Fortunately, owing to operation of selection rules, cases of high isomer ratios are known.

Third: Since the Mössbauer line must not be greatly broadened if lasing is to ensue, the kinetics of resonance involves an inertial time-lag of the order of the reciprocal linewidth.

In short, an accurate account of the kinetics of a neutron-pumped graser involves the history of neutron generation, moderation, and capture, as well as of the lasing process. Baldwin and Scydam have worked out the kinetics for a model in which the moderator is hydrogen, assumed infinite, the source is a spatially uniform delta function, the neutrons undergo resonance capture, and the Mossbauer line is of an arbitrarily assigned breadth.

FIGURE 10

As an example of the application of our kinetic model, we consider the Mössbauer isomer Kr-83, which emits a 9.3-keV gamma ray from a 147-ns excited state. In Mössbauer experiments, this state is formed by beta decay; we form it by neutron capture in Kr-82. The radiative transition is internally converted, so that only about 5% of the excited states decay to the ground state by recoilless gamma radiation under the best of conditions. Moreover, a 1.86-h metastable level is populated in 2/3 of all neutron capture events. Other properties are favorable, however.

FIGURE 11

The computer study of Kr-83 (by Laurie McNeil) accepted a set of arbitrary input parameters: e.g., source neutron density, density of the lasing medium, isomer ratio, neutron capture parameters and Debye temperature. It calculated the time dependence of the amplitude of an electromagnetic wave, initiated at the origin by spontaneous emission coincident with the neutron burst, and also

of the parent and laser state populations. The parameter in the upper curve is distance from the spontaneous source in nonresonant absorption lengths. For this case, we have nearly pure exponential attenuation and decay; there is only slight amplification after about one-and-one-half mean lifetimes. The neutron source intensity was nevertheless quite high: $5(20)$ neutrons cm^{-2} . Approximately 5% of the parent population has been depleted by about 2.5 mean lifetimes.

FIGURE 12

By incorporating the Kr in a Be host at very low temperature and increasing the source-burst intensity to $1(22)$ cm^{-2} , we now see amplification; still, however, barely enough to overcome the nonresonant absorption. The delay is mainly the time for neutrons to moderate to the 40-eV capture resonance of Kr-82. Further moderation to thermal energy accomplishes little, however, since the parent population by then is practically depleted, and accumulation of ground states overcomes excited state formation.

Similar results are found for other isotopes. Note that this simplified model has neglected the effect of neutron bombardment of the graser states; more important, it has not questioned the possibility that so many neutrons can indeed be generated and moderated -- more on this later.

FIGURE 13

We therefore turn to the two-stage proposal. In it, the graser levels are pumped by Mössbauer radiation, generated in a large volume of converter and resonantly absorbed in the graser.

FIGURE 14

Since resonance absorption can only saturate, not invert, a three-level scheme is essential; viz., the isotope Dy-161 has several Mössbauer lines and a stable parent isotope, Dy-160, from which it can be formed by neutron capture. Baldwin and Solem list four such cases.

FIGURE 15

The kinetics of the two-stage system is, of course, more complicated than

that of the directly-pumped system. As a beginning, we neglect the time for neutron moderation and assume capture, at a rate " γ " per second in the parent, to be a step function, populating the three levels in fixed ratios. The two excited levels, of course, have distinct decay constants " λ ". Level 2 is populated both directly by neutron capture and by decay of level 3. The three gamma radiations are attenuated by nonresonant absorption in the convertor and by resonant scattering in the graser as well. The net pumping rate coefficients W_{ij} in the graser are related to the emission rates in the convertor by coupling coefficients K_{ij} that are calculated as follows:

FIGURE 16

First, the time-dependent sources of recoilless radiation in the convertor, excited by neutron capture, are determined.

FIGURE 17

Next, the vector flux of each respective radiation at the convertor-graser interface is calculated, taking account of obliquity and nonresonant absorption.

FIGURE 18

Note that these fluxes are dependent upon internal conversion, parent depletion, isomer ratios, and recoilless fraction.

The effect of resonant scattering in the graser is to replace the normally exponential attenuation with distance by a more complicated dependence on both distance and time, since the cross section for resonant interaction is time-dependent unless the line is greatly broadened. Here we optimistically assume that it is of natural width.

FIGURE 19

Mössbauer radiation is absorbed in the graser by both nonresonant and resonant processes. Only the latter excite the nucleus. Near the interface, the probability of resonant absorption is found to contain three factors: 1) a geometrical factor, which for a nearly plane interface is $1/4$; 2) the ratio of resonant to nonresonant absorption coefficients -- this, it will be recalled, was the basic advantage claimed for two-stage pumping; 3) a kinetic factor, embodying the inertial time-lag of resonance scattering.

Finally, we refer the probability for excitation of a transition to the total nuclear population of ground states in the graser medium, $W(t)$.

FIGURE 20

These pumping rate coefficients can now be inserted in rate equations for the populations of the three states. Note that, unlike direct pumping, two-stage pumping recycles the active nuclei in the graser; depletion still occurs in the convertor, of course. Each level is formed by direct absorption or by decay of a higher level, and destroyed by its decay or by resonant absorption of an appropriate radiation.

FIGURE 21

The case of Ge-73 illustrates the results. The ordinate is degree of excitation: positive above inversion, negative below. The abscissa is the logarithm of the neutron capture rate coefficient (" γ " in earlier figures), a product of neutron flux by capture cross section of the parent. If only the lower level were pumped, we would have the curve 2.1 which, of course, can saturate but not invert the populations. If both excited states are pumped, we can indeed invert Ge-73 in principle, provided the neutron flux is high enough to give a capture rate of $1(6) \text{ s}^{-1}$.

FIGURE 22

Unfortunately, for this isotope, the coupling coefficients (K_{ij}) are so low that there is negligible advantage from two-stage pumping. Moreover, although the 2-1 transition can be inverted, it cannot lase, because its nonresonant absorption cross section is too high (unless the radiation is anomalously transmitted). Finally, the neutron capture cross section of Ge-72 is only about 1 barn, so the neutron flux must exceed $1(30) \text{ cm}^{-2} \text{ s}^{-1}$ for inversion.

FIGURE 23

Having found no known isomer with encouraging properties, we invented one. For this hypothetical isomer, the capture rate coefficient must be only $1(4) \text{ s}^{-1}$.

FIGURE 24

Being at liberty to choose any properties within reason for this hypothetical isotope, we assumed no internal conversion, full recoilless emission fraction, and, most optimistically, a neutron capture cross section equal to the highest known, 2.6 megabarns. As a result, we now need only about $6(21)$ neutrons $\text{cm}^{-2} \text{ s}^{-1}$ to invert this imaginary transition. Such a high cross section can be realized only with fully thermalized neutrons, velocity $2.2(5) \text{ cm s}^{-1}$. The corresponding neutron density at the time of capture must therefore be at least $1(16) \text{ cm}^{-3}$.

Had we considered the kinetics of neutron moderation, as in the directly pumped cases, an appreciably higher threshold would have been found. Moreover, other simplifying assumptions in this analysis all have the effect of underestimating the excitation requirement. Neutron pumping is clearly going to require an enormous density of moderated neutrons.

Can we, in fact, produce it, even with a nuclear explosion?

FIGURE 25

The answer is that we cannot. Baldwin and Solem have shown that to assume otherwise is to violate not just the facts of recent experience, but the laws of thermodynamics.

Let us inject a density n_0 of fast neutrons into an infinite moderator (so that leakage and diffusive losses are neglected). The energy of the neutrons, imparted to the atoms of the moderator, raises their temperature. According to the Second Law, when the temperatures of the neutron gas and the moderator have become equal, moderation must cease. A straightforward energy balance calculation (i.e., applying the First Law) shows that no more than $3.3(18)$ fission neutrons can be moderated to 200 eV, and a far smaller number to thermal energies.

FIGURE 26

When diffusion and leakage are taken into account, the situation is even less favorable. This figure shows the maximum density allowed by energy balance in a finite, heavy atom moderator. For example, in a Pb moderator, of 50 cm radius, no more than $1(13)$ neutrons cm^{-3} can be moderated to 200 eV, regardless of the source burst strength. The cross in the figure is the Goldanskii-Kagan estimate for Ta-181; the circle, McNeill's computer result for Kr-83.

FIGURE 27

It is unlikely that transient features of heat conduction can modify this result. Here we show the calculated time-dependences of 2-MeV, 50-keV, and 1-keV neutrons after a delta-function source burst. Most of the heating is, of course, done by the fastest group of neutrons, and is accomplished before the lowest-energy group has been populated.

FIGURE 28

Given these limitations on neutron density, can we pump a graser?

Here we again consider a hypothetical isomer, with 200 eV capture resonance and a mean lifetime closely matching the time for neutrons to moderate to thermal energies in hydrogen or to the capture resonance in a heavy moderator, Pb. The pumping rate coefficients turn out to be about the same as those estimated above for the other hypothetical isomer. The actual fraction transmuted is very low, indeed.

We do not expect any actual isomer to begin to approach even so unfavorable a result.

FIGURE 29

While we are therefore forced to conclude that in-situ pumping of a graser by a pulsed neutron source is not feasible, we hasten to add that there are other, as-yet-unexplored proposals, based on longer-lived transitions that might be excited separately, then purified and assembled. These, too, have their problems; particularly that of linebroadening - in real solids. At present, no sufficiently detailed examination of these proposals has appeared to warrant a categorical conclusion; we can say only that these will be difficult at best.

X-RAY vs γ -RAY LASERS?

FOR LASING

$$N^* \sigma > \mu$$

BUT

$$\begin{cases} \sigma \sim \Lambda^2 \\ \mu \sim \Lambda^3 \end{cases}$$

CRITICAL EXCITATION DENSITY $N^* \sim \Lambda$

PUMP RATE

$$R \sim N^* / \tau \sim \Lambda / \tau$$

PUMP POWER

$$P > h\nu_p \cdot R \sim 1/\tau$$

LIFETIMES:

ATOMIC

$$\tau_K \sim 1(-16) \Lambda$$

$$P \sim 1(17) \text{ W cm}^{-3}$$

NUCLEAR

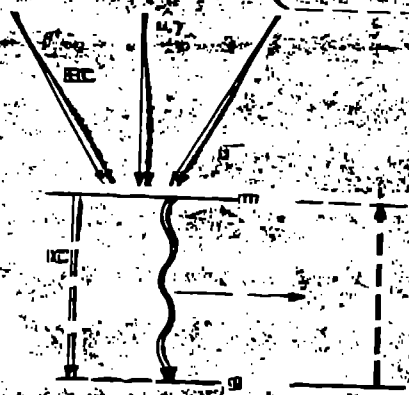
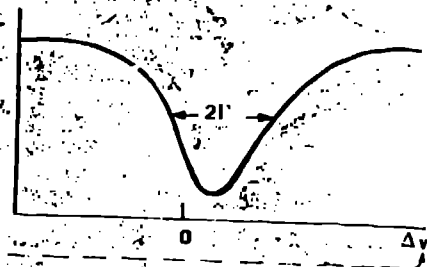
$$1(-12) < \tau < 1(10)$$

$$@ \Lambda = 1 \text{ \AA}$$

!!!

W

MOSSBAUER EFFECT



$$G_R = \frac{\lambda^2}{2\pi} \frac{f}{1+a} \frac{1}{\Gamma}$$

IN SOLIDS

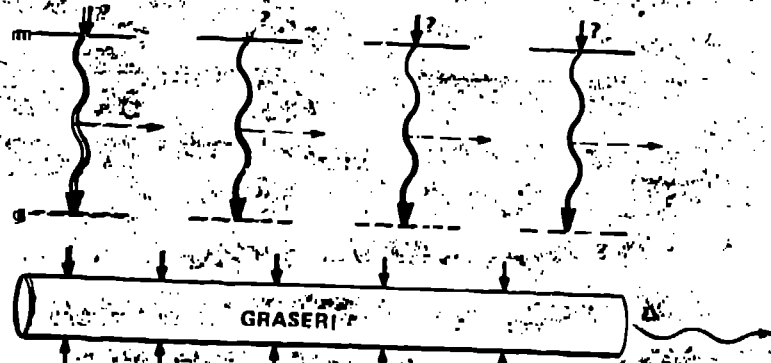
$$f = f(E_\gamma, \theta, T) \approx 1$$

$$\Gamma \approx 1$$

$$G_R \gg G_{sm}$$

SUGGESTS

STIMULATED EMISSION



LA

GOL'DANSKII-KAGAN JETP 37, 49 (1973)

PROPOSE: PUMP TO INVERSION BY CAPTURE OF NEUTRONS
FROM A "NUCLEAR EXPLOSION "

FOR RECOILLESS EMISSION, REQUIRE : 10^{-4} s FOR $\Gamma < 10/\tau$

$$\left. \begin{array}{l} E = 100 \text{ keV} \\ T = 10^{-4} \end{array} \right\} \tau \sim 1$$

PROBLEMS

- NON-RESONANT ABSORPTION
- HEATING BY RECOIL FROM SCATTERING
RECOIL FROM CAPTURE
CAPTURE γ -RAY ABSORPTION

SUGGESTED SOLUTION

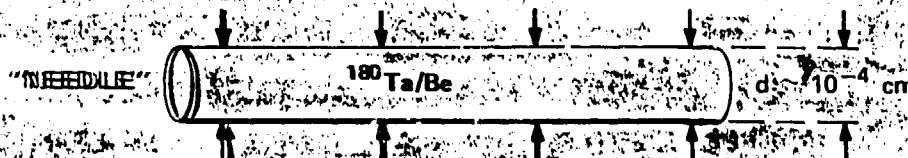
1. BE HOST, INERT ACTIVE MATERIAL

- HIGH CAPACITY
- HIGH ρ
- LOW γ ABSORPTION
- LOW σ CAPTURE

2. FILAMENT GEOMETRY (d = 10 μ)

- ALLOWS ESCAPE OF RADIATION
- DEFINES BEAM

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HEATING?WE REQUIRE

$$N = \frac{\mu}{\sigma_R} = \frac{2.8 (18)}{17} \text{ CAPTURES cm}^{-3}$$

RELEASING

$$8 \text{ MeV } (f_n) = \frac{E_{\text{IN}}}{A} \text{ (RECOIL)}$$

FILAMENT GEOMETRY

AVERAGE DISTANCE TO SURFACE $\pi d/4 \sim 8 (-5) \text{ cm}$

X

ABSORPTION COEFFICIENT $\mu \sim 6.1 \text{ cm}^{-1}$

FRACTION OF CAPTURE \rightarrow ENERGY ABSORBED $< 5 (-4)$

TEMPERATURE RISE

(ENERGY ABSORBED PER ATOM)

$$< 5 (-4) \times 8 (6) \frac{\text{eV}}{\text{CAPTURE}} \times 2.8 (18) \text{ CAPTURES} \times \frac{1}{1.2 (23) \text{ ATOMS}}$$

$$< 0.083 \text{ eV/ATOM}$$

$$< 900^\circ \text{ K}$$

PROVIDED RECOIL IS NEGLIGIBLE (LOW E_N)

LL

NEUTRON BURST REQUIREMENT

WE REQUIRE

$$(N_m - N_g) > \mu / \sigma_R \quad \text{FOR GRASER}$$

SO

$$N_C > N_m + N_g > \frac{0.1 N_P}{\eta \tau} \quad \text{CAPTURES cm}^{-3}$$

$$\text{WITHIN } \Delta t \ll \tau = 10 \mu s$$

FLUENCE

$$N_C = N_P \sigma_C \int \phi dt$$

$$\int \phi dt = \frac{N_C}{N_P \sigma_C} > \frac{0.1}{\eta \tau \sigma_C} \quad \text{NEUTRONS cm}^{-2}$$

G-K ESTIMATE FOR ^{180}Ta

$$\sigma_C = 17000 \cdot \sqrt{\frac{0.025}{E_n}} \quad (-24) \text{ cm}^2$$

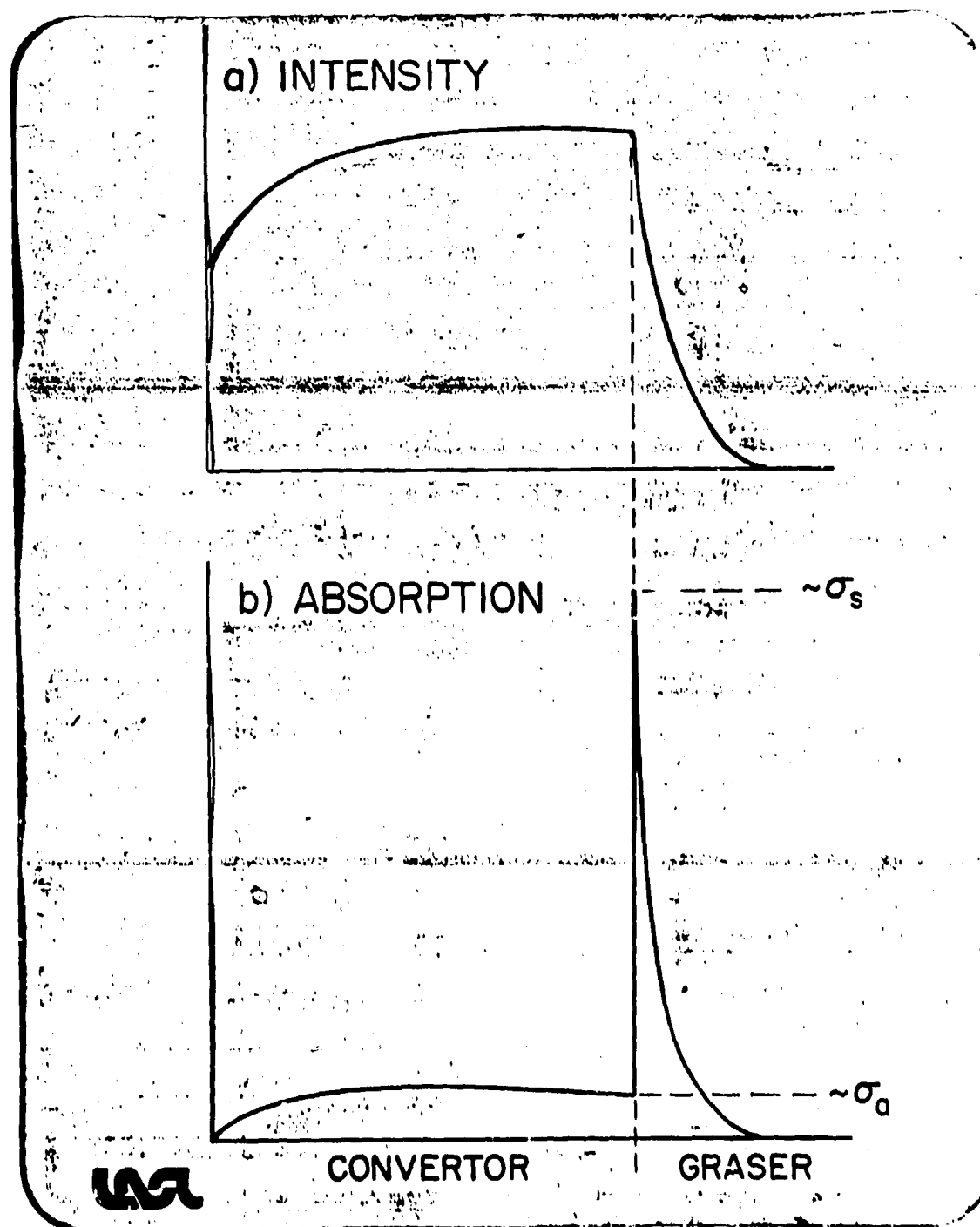
$$\int \phi dt \sim n v \tau > 4 (20) \sqrt{E_n} \text{ cm}^2$$

$$\text{FOR } E_n < 100 \text{ eV}, v = 1.4 (7) \text{ cm s}^{-1}$$

THEY REQUIRE

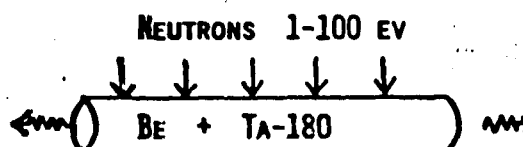
$$n > \frac{4 (20) \sqrt{100}}{1.4 (7) \times 10^{-6}} = 3 (19) \text{ cm}^{-3}$$

LSA

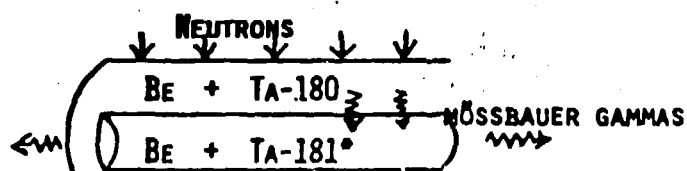


TYPE 1 GRASERS (PUMPED IN SITU)

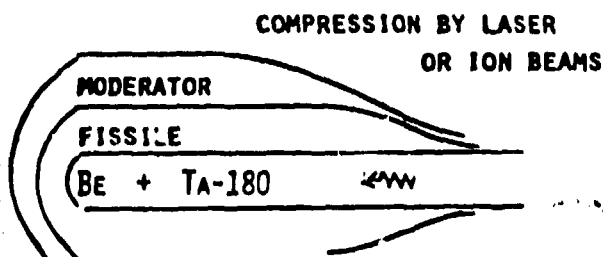
1) DIRECT EXCITATION (GOL'DANSKII & KAGAN; SOLEM)



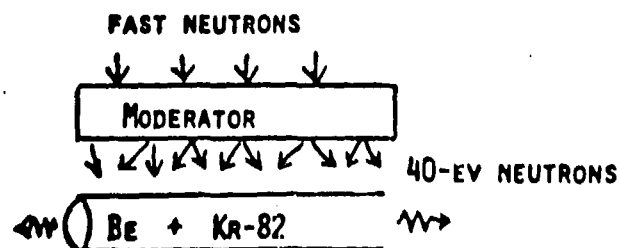
2) TWO-STAGE EXCITATION (GOL'DANSKII, KAGAN & KAMIOT)



3) TRAVELING WAVE EXCITATION (BALDWIN)



4) MOVING MODERATOR, RESONANCE CAPTURE EXCITATION (BOWMAN)



WJ

RECENT GRASER STUDIES AT LASL

"KINETICS OF STIMULATED EMISSION IN NEUTRON-PUMPED NUCLEAR LASER SYSTEMS"

G. C. Baldwin, in LASER INTERACTION AND RELATED PLASMA PHENOMENA 4A, 259-256 (1977).

"KINETICS OF NEUTRON-BURST-PUMPED MOSSBAUER GAMMA-RAY LASERS"

G. C. Baldwin and R. R. Suydam, LASL Report LA-UR-77-140 (1977).

"KINETICS OF STIMULATED EMISSION IN NEUTRON-PUMPED ^{83}Kr "

G. C. Baldwin and L. R. McNeil, LA-7004-MS (1977).

"A THEORETICAL ASSESSMENT OF RADIATION DAMAGE IN A PROPOSED GAMMA-RAY LASER"

H. R. Schwann, RPI MS Thesis, published as LA-7009-T (1978).

"MAXIMUM DENSITY AND CAPTURE RATE OF NEUTRONS MODERATED FROM A PULSED SOURCE"

G. C. Baldwin and J. C. Solem, Nuc. Sci. and Eng'g 72, 281-289 (1979).

"THE DIRECT PUMPING OF GAMMA-RAY LASERS BY NEUTRON CAPTURE"

G. C. Baldwin and J. C. Solem, Nuc. Sci. and Eng'g 72, 290-292 (1979).

"TWO-STAGE PUMPING OF THREE-LEVEL MOSSBAUER GAMMA-RAY LASERS"

G. C. Baldwin and J. C. Solem, Jour. Appl. Phys. 51, 2372-2380 (1980).

"TIME-DOMAIN SPECTROSCOPY OF RECOILLESS GAMMA RAYS"

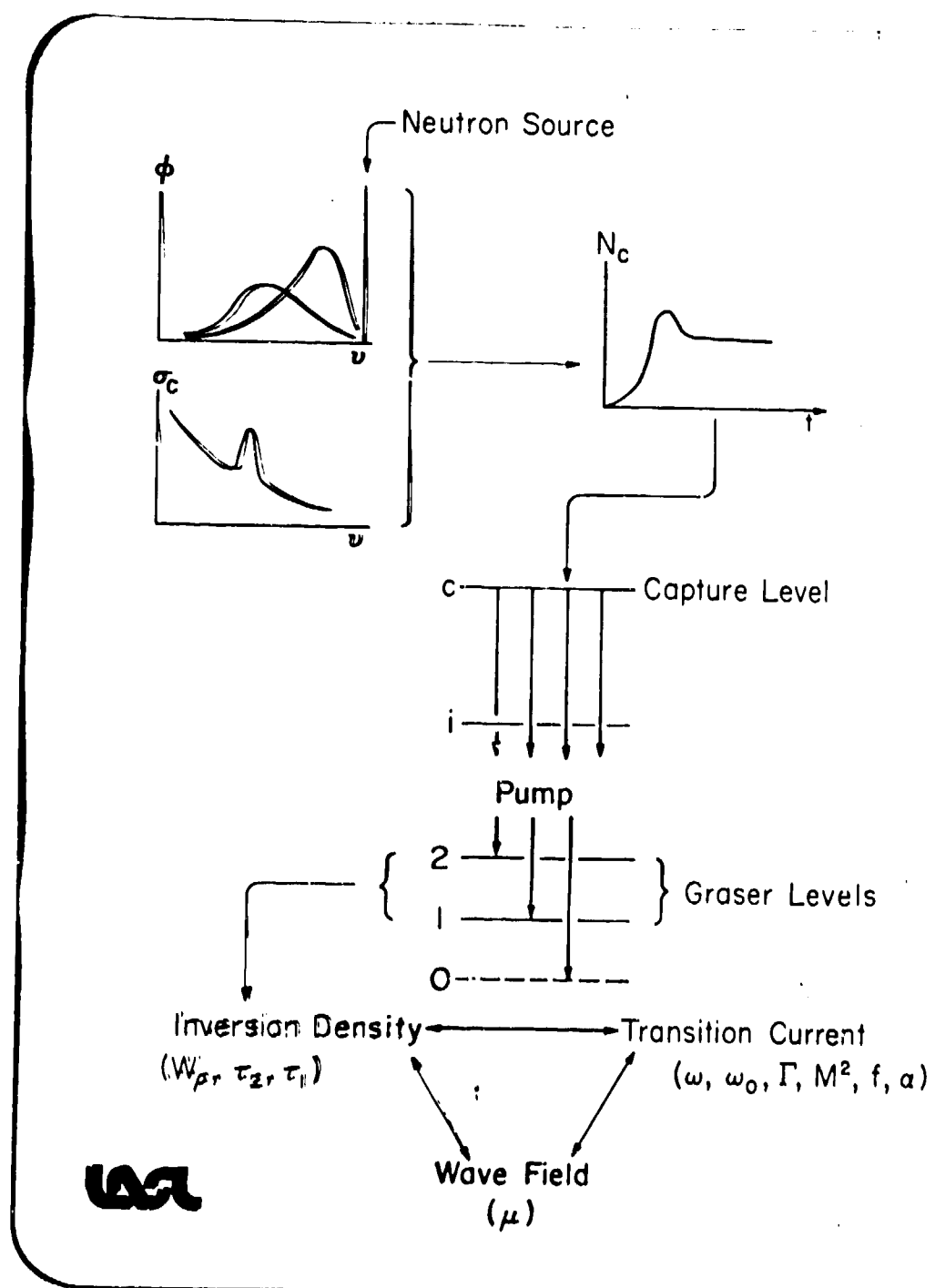
G. C. Baldwin, Nuc. Instr. and Methods 159, 309-330 (1979).

"CORRELATION COUNTING IN TIME-DOMAIN MOSSBAUER SPECTROSCOPY"

G. C. Baldwin and V. I. Gol'danskii, Nuc. Instr. and Methods 169, 581-583 (1980).

LA

8



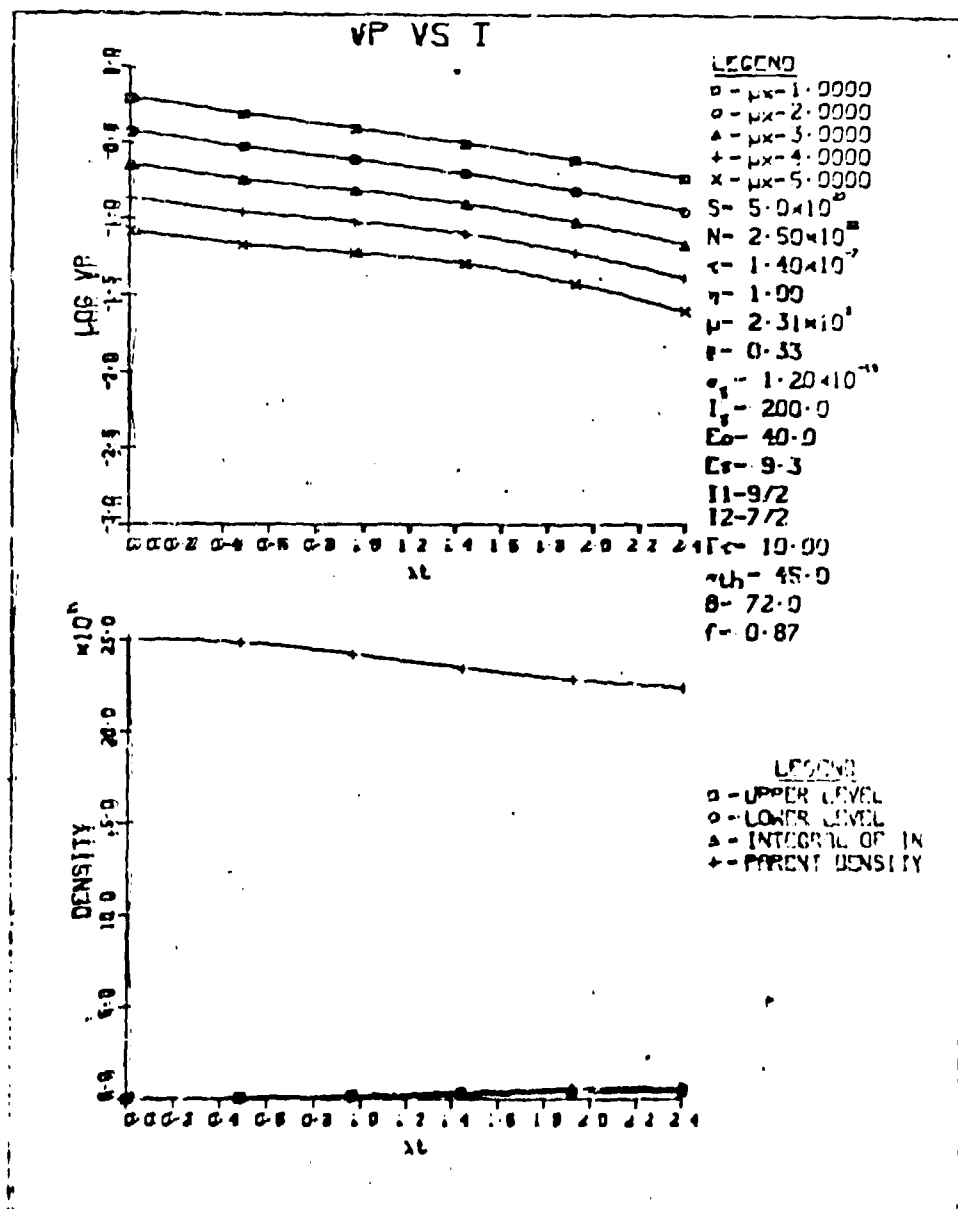


Fig. 1 - S = 5.0(20), pure krypton, tenfold-broadened line, T = 0.

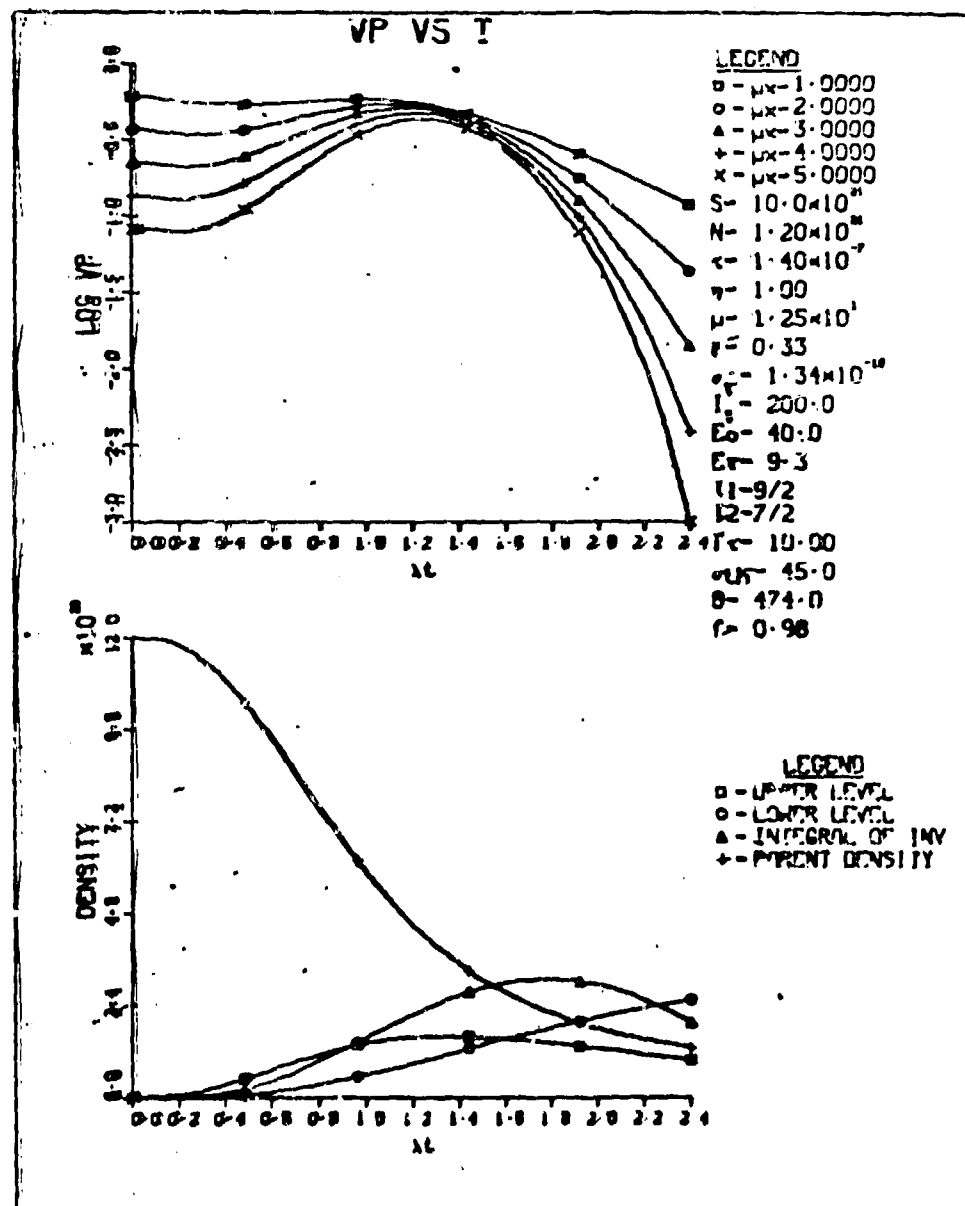
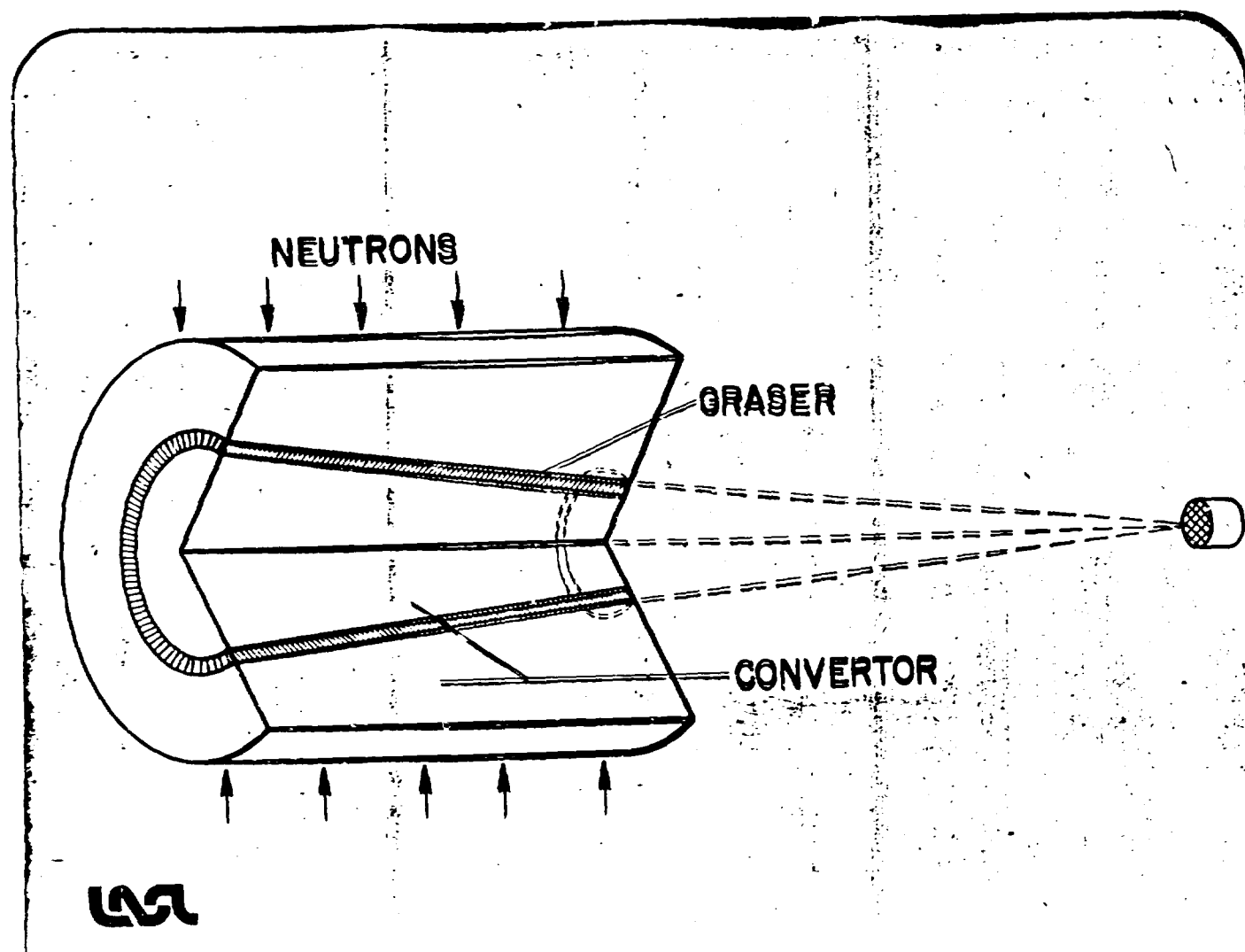
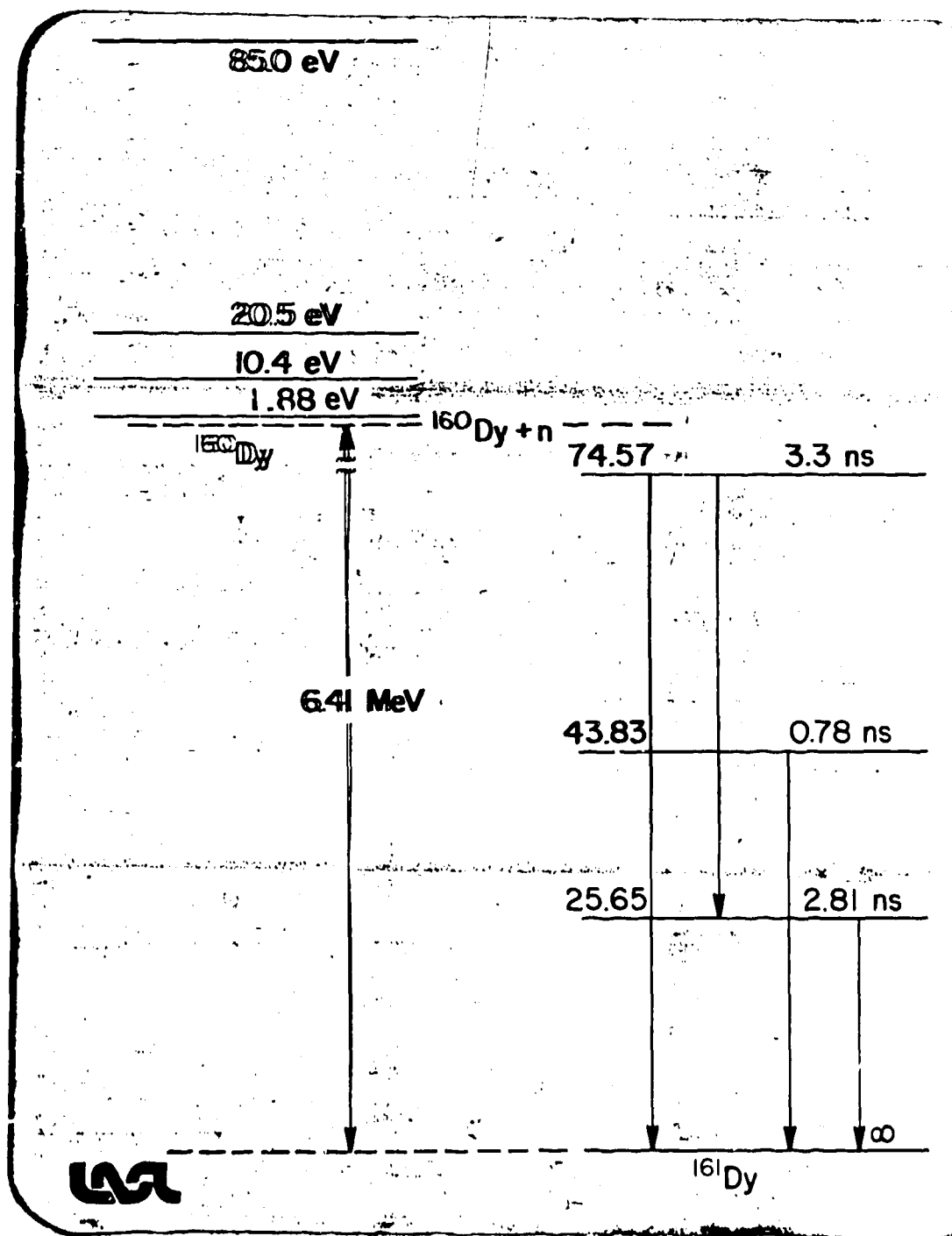
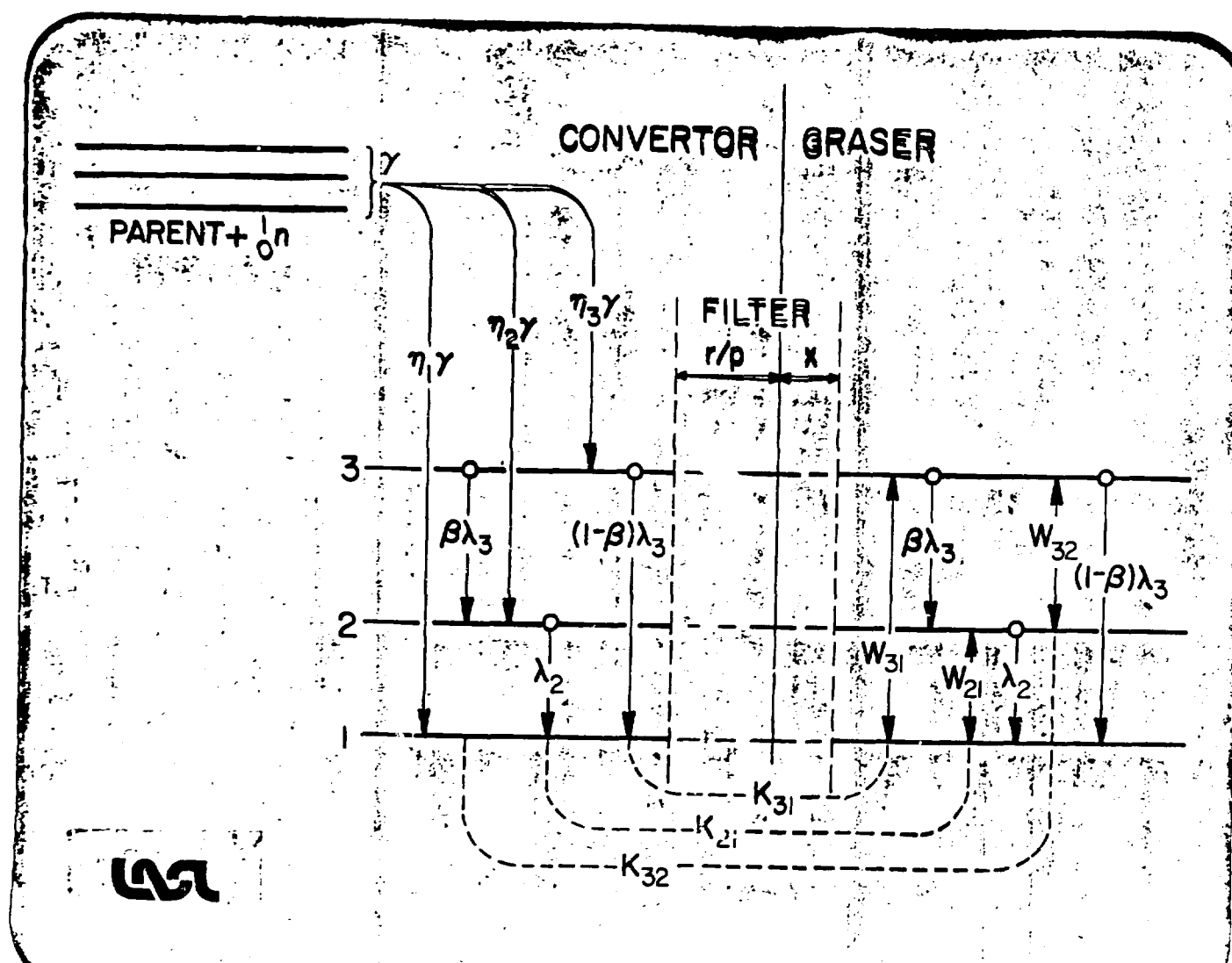


Fig. 8 - $\delta = 1.0(22)$, 1:100 Kr:Se, tenfold-broadened line, $T = 0$.





INTERSTAGE COORDINATING SCHEM
FOR A TWO-STAGE, THREE-LEVEL
GRASER SYSTEM



THREE-LEVEL CONVERTOR SOURCES

$\text{em} = 3 \text{ s}^{-1}$

$$S_{31} = N_o \gamma \frac{\eta_3 \lambda_3 (1 - \beta) f_{13}}{1 + \alpha_{13}} \left[\frac{e^{-\gamma t} - e^{-\lambda_3 t}}{\lambda_3 - \gamma} \right]$$

$$S_{32} = N_o \gamma \frac{\eta_3 \lambda_3 \beta f_{23}}{1 + \alpha_{23}} \left[\frac{e^{-\gamma t} - e^{-\lambda_3 t}}{\lambda_3 - \gamma} \right]$$

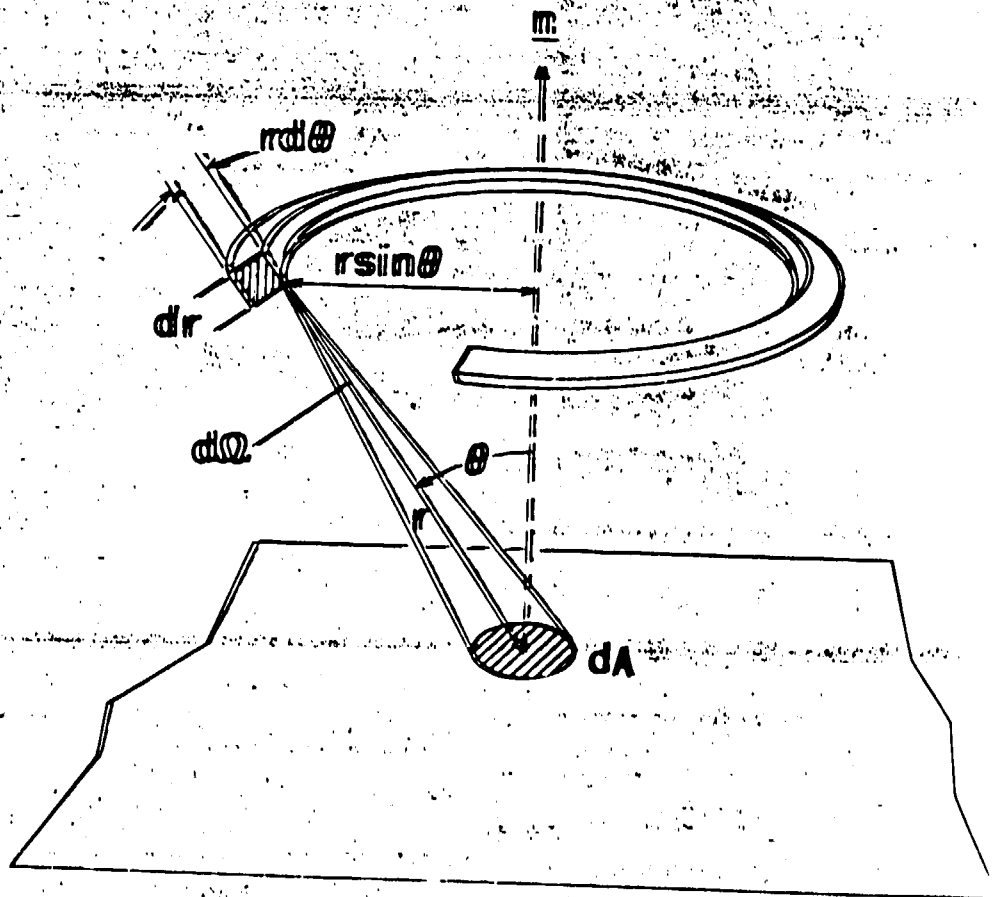
$$S_{21} = N_o \gamma \frac{\lambda_2 f_{12}}{1 + \alpha_{12}} \left[\eta_2 \left(\frac{e^{-\gamma t} - e^{-\lambda_2 t}}{\lambda_2 - \gamma} \right) + \eta_3 \beta \frac{\lambda_3}{\lambda_3 - \gamma} \left(\frac{e^{-\gamma t}}{\lambda_2 - \gamma} - \frac{e^{-\lambda_3 t}}{\lambda_2 - \lambda_3} \right) \right]$$

UN

$$dV = 2\pi r^2 \sin\theta \, dr \, d\theta$$

$$d\Omega = dA \cos\theta / 4\pi r^2$$

$$\frac{d^2\psi}{d\Omega dA} = \frac{1}{2} \int_0^\pi dr \left(e^{-\mu r} S(r) \sin\theta \cos\theta \right)$$



LA

RECOILLESS FLUX AT INTERFACE $\text{cm}^{-2} \text{s}^{-1}$

$$\frac{d^2 \psi}{dA} = - \frac{S(t')}{2\mu} p dp \quad p = \cos \theta$$

WHERE

$$S(t') = \frac{\gamma N_p \eta \beta \pi}{\pi + \alpha} \text{cm}^{-3} \text{s}^{-1}$$

$$\gamma = \Phi \sigma_c \quad N_p = N_0 \exp(-\gamma t)$$

EFFECT OF RESONANT FILTER

$$G(x(p, t, t')) = \lambda \exp \left\{ -\frac{\mu x}{p} - \lambda (t - t') \right\} J_0^2 \left| \sqrt{\frac{\kappa \lambda x (t - t')}{p}} \right|$$

WHERE

$$\kappa_{ij} = [N_i (g_i/g_j) - N_j] \sigma_s \text{cm}^{-1}$$

$$\sigma_s = \frac{2.45 (-15) \beta f}{E^2 (1 + \alpha)} \text{cm}^2$$

W

ABSORPTION RATE IN GRASER

 $\text{cm}^{-3} \text{s}^{-1}$

$$P_{\text{as}}(k_x, t) = - \frac{\partial}{\partial x} \int_0^\pi \left(\frac{d^2 \psi}{d\lambda d\mu} \right) G(x, t - t') dp$$

$$= \frac{\lambda}{2\mu} \int_0^\pi p dp \int_0^\pi \exp \{ -\lambda (t - t') \} S(t') \frac{\partial}{\partial x} \left[\exp \left\{ -\frac{\mu x}{p} \right\} \right] J_0 \left\{ \frac{1}{p} \right\}$$

$$\text{WHERE } \frac{1}{\rho} = \sqrt{\frac{HOK\lambda (t - t')}{\rho}}$$

$$= P_{\text{nonresonant}} + P_{\text{resonant}}$$

$$P_{\text{res}}(0, t) = \frac{\pi}{4} \cdot \frac{H}{\mu} \cdot \lambda^2 \exp \{ -\lambda t \} \int_0^\pi (t - t') S(t') \exp \{ \lambda t' \} dt'$$

"Kinetic factor"
 "absorption factor"
 "geometrical factor"

INDUCED TRANSITION COEFFICIENT, $x \rightarrow 0$

$$W(t) = \frac{1}{N_0} \left(\frac{\sigma_s}{4 \sigma_0} \right) \cdot (\text{KINETIC FACTOR}) \cdot t^{-1}$$

GRASER PUMPING RATE COEFFICIENTS

$$W_{31} = K_{31} \gamma \Rightarrow \gamma \frac{\eta_3 (1 - \beta) f_{13}}{1 + a_{13}} \left(\frac{\theta_1}{4\theta_3} \right)_{31} \quad \theta = 1$$

$$W_{32} = K_{32} \gamma \Rightarrow \gamma \frac{\eta_2 \beta f_{23}}{1 + a_{23}} \left(\frac{\theta_1}{4\theta_3} \right)_{23} \quad \theta = 1$$

$$W_{21} = K_{21} \gamma \Rightarrow \gamma \frac{(\eta_2 + \beta \eta_3) f_{12}}{1 + a_{12}} \left(\frac{\theta_1}{4\theta_3} \right)_{21} \quad \theta = 1$$

RATE EQUATIONS FOR POPULATION DENSITIES

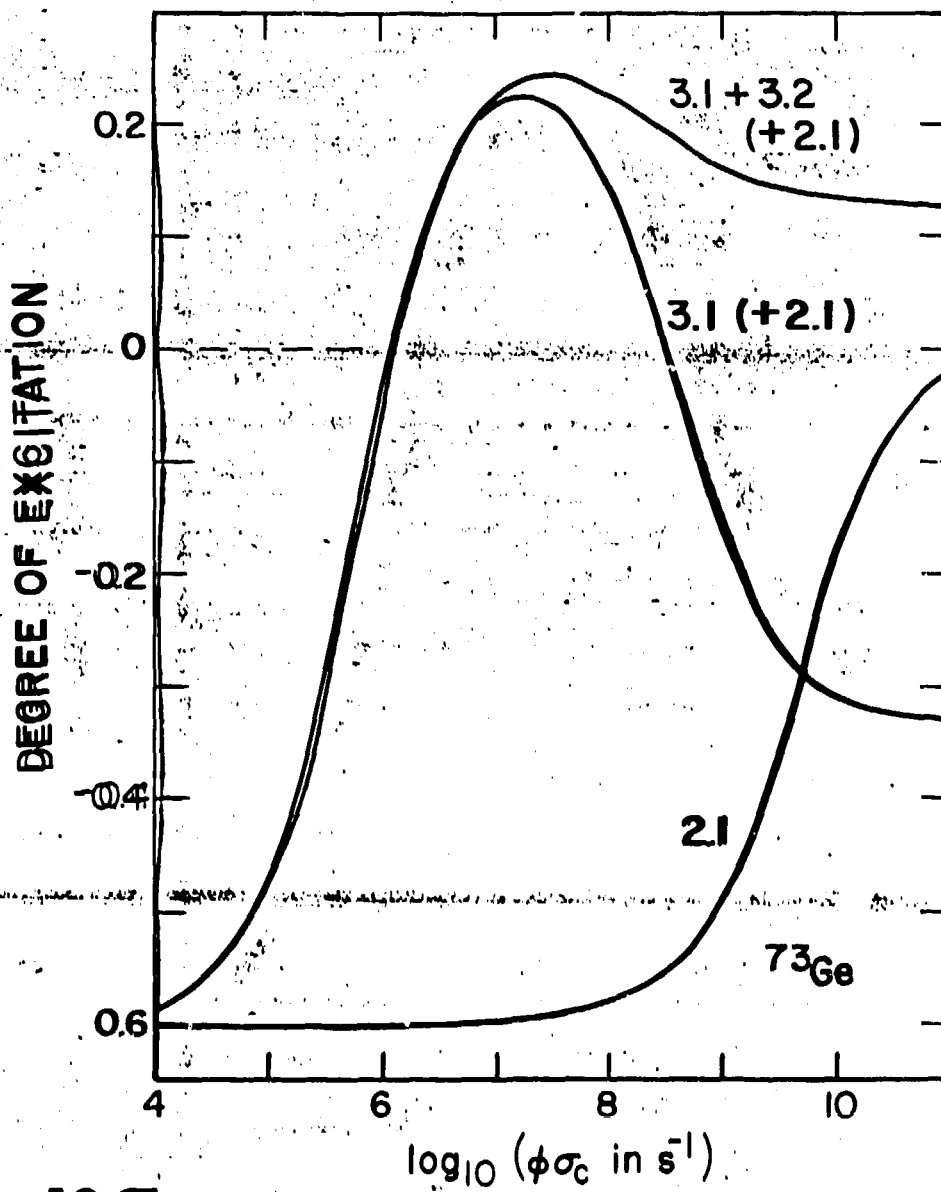
$$N_0 = N_1 + N_2 + N_3$$

$$\dot{N}_2 = N_1 W_{21} \left[\frac{\theta_2}{\theta_1} \right] + N_2 \left(W_{32} \left[\frac{\theta_2}{\theta_3} \right] - W_{21} + \lambda_2 \right) + N_3 \left(W_{32} + \beta \lambda_3 \right)$$

$$\dot{N}_1 = -N_1 \left(W_{21} \left[\frac{\theta_2}{\theta_1} \right] + W_{31} \left[\frac{\theta_3}{\theta_1} \right] \right) + N_2 \left(W_{21} + \lambda_2 \right) + N_3 \left(W_{31} + [1 - \beta] \lambda_3 \right)$$

$$N_1(0) = N_0$$

$$N_2(0) = N_3(0) = 0$$



UN

^{73}Ge

<u>LEVEL</u>	<u>E</u>	<u>θ</u>	<u>η</u>	<u>λ</u>
3	68.8	8	0.14	3.7(8)
2	13.3	6	0.12	2.4(5)

<u>TRANSITION</u>	<u>E</u>	<u>f</u>	<u>α</u>	<u>σ_s</u>	<u>σ_B</u>	<u>K</u>
3-1	68.8	0.10	0.81	5.4 (4)	171	0.54
3-2 ($\beta = 0.53$)	55.5	0.30	0.15	1.85(4)	301	0.48
2-1	13.3	0.01	1.1(3)	8.0 (3)	1.5(4)	3.2 (-5)

$\sigma_c = 0.98 \text{ b.}$

INVERSION FOR $\phi > 1 \text{ (30) cm}^{-2} \text{ s}^{-1}$



PARAMETERS OF A HYPOTHETICAL
IDEAL ISOMER FOR TWO-STAGE
THREE-LEVEL PUMPING

M? Hy
Z?

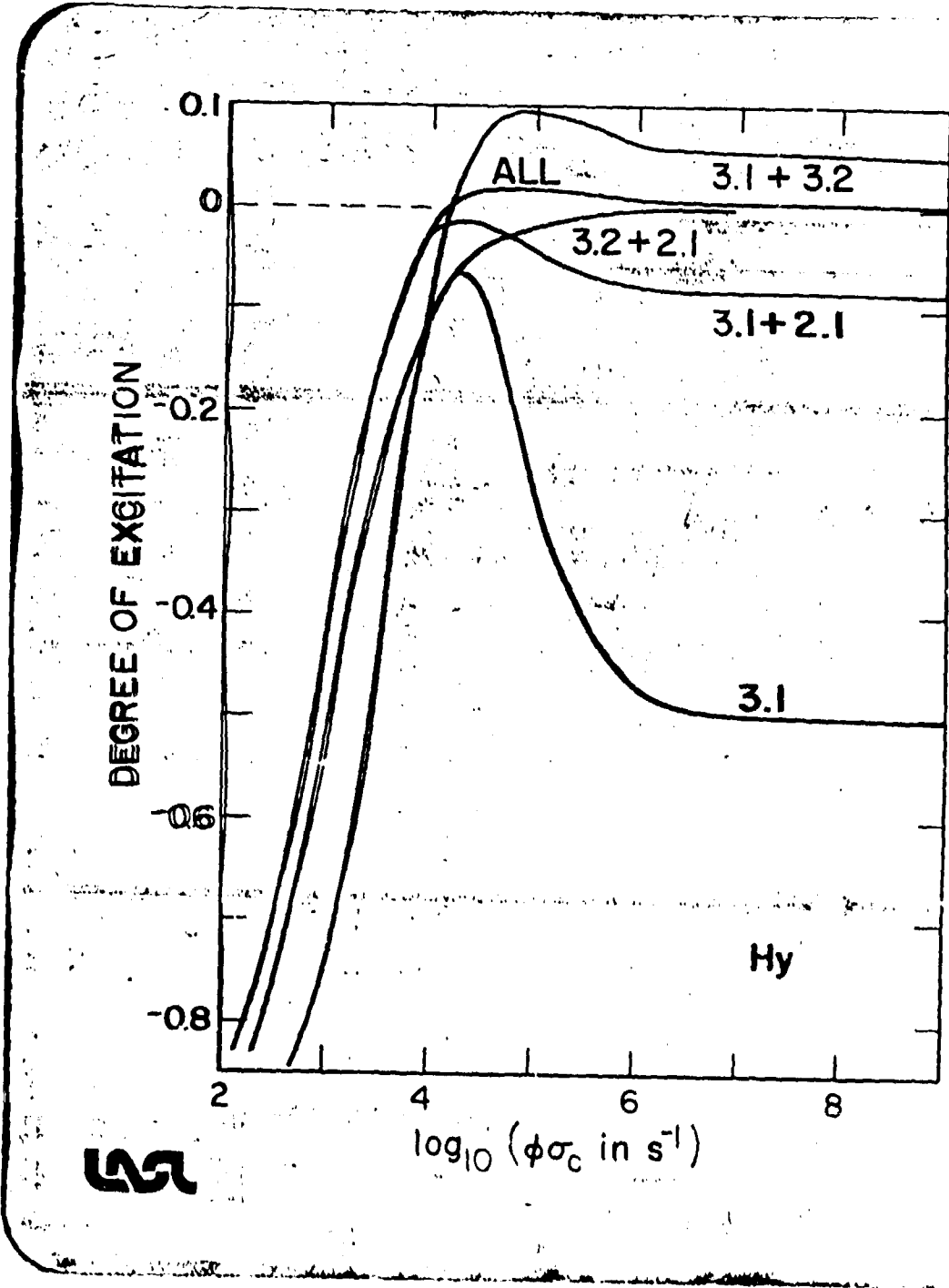
LEVEL	E	g	η	λ
3	30	1	0.33	1.0(6)
2	15	1	0.33	1.0(5)

TRANSITION	ΔE	f	α	σ_s	σ_a	K
3-1	30	1.0	0.0	1.0(7)	9.1(3)	18.1
3-2 ($\beta=0.8$)	15	1.0	0.0	2.7(6)	8.3(3)	21.5
2-1	15	1.0	0.0	2.7(6)	8.3(3)	48.3

$\sigma_c = 2.6 (6) \text{ b}$

INVERSION THRESHOLD $\phi = 5.8(21) \text{ cm}^{-2} \text{ s}^{-1}$

UN



UNIFORM INJECTION INTO INFINITE MODERATOR

NEUTRON DENSITY

$$n_0 = n_0(E_0) = n(E) \text{ NEUTRONS cm}^{-3}$$

MODERATOR TEMPERATURE

$$T_1(E) = n_0(E_0 - E)/N \quad (\text{FIRST LAW})$$

$$T(E) < E \quad (\text{SECOND LAW})$$

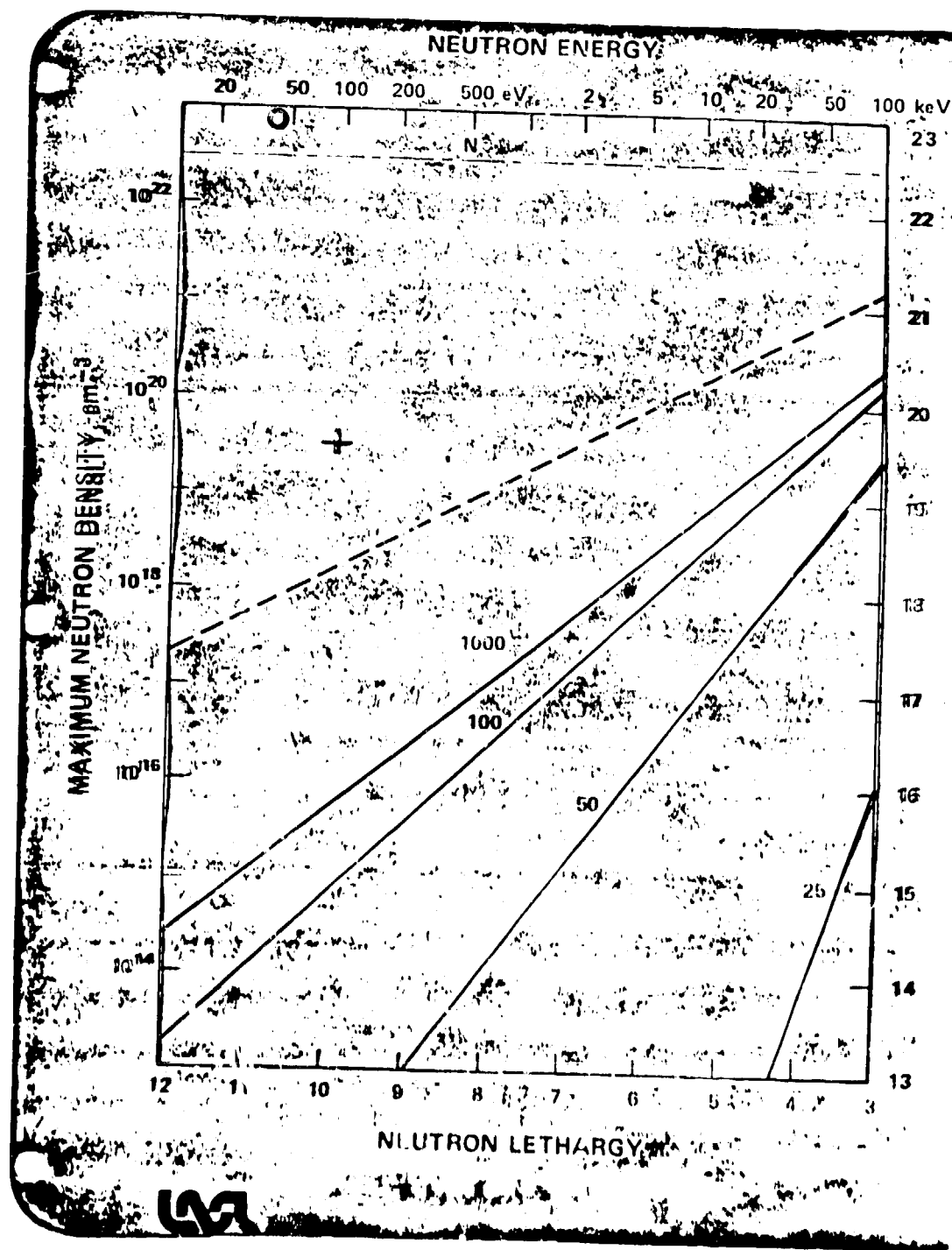
$$n(E) < NE/(E_0 - E) \sim N(E/E_0)$$

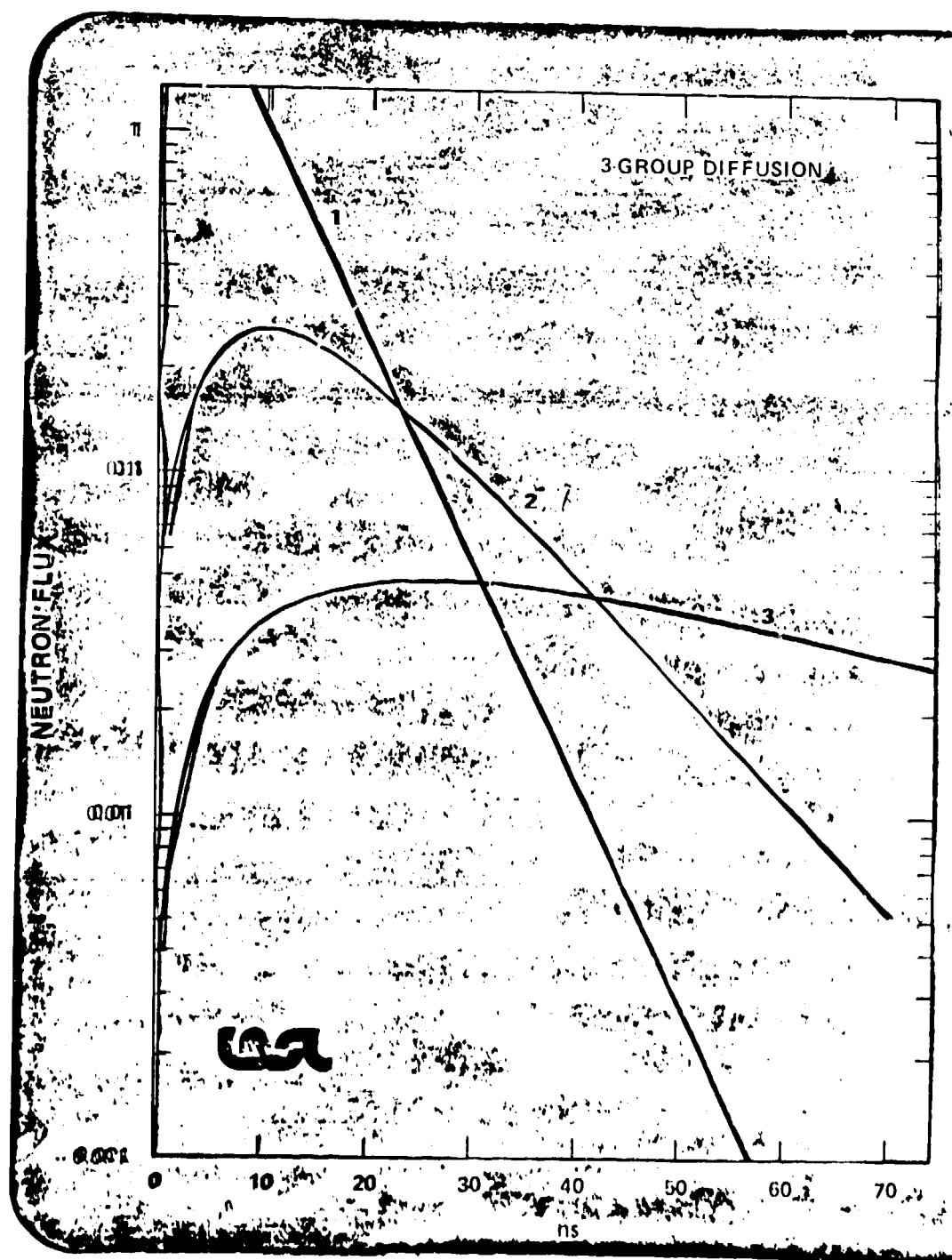
e.g. $E_0 = 2.6 \text{ eV}$ (FISSION)

$E = 200 \text{ eV}$ (RESONANCE)

$N = 3.3(22) \text{ cm}^{-3}$ (Pb)

$n \sim 3.3(18) \text{ cm}^{-3}$





NEUTRON PUMPING OF Mg ($\tau = 10 \mu s$, $E_f = 200.0 V$)

MODERATOR	<u>H</u>	<u>Pb</u>	
NUMBER DENSITY	0.7 (22)	3.3 (22)	cm^{-3}
LETHARGY GAIN PER COLLISION	1.0	0.0096	—
MODERATION TIME	0.4 (-9)	29 (-6)	s
MODERATION RATE AT E_f	2 (10)	1.4 (7)	$eV s^{-1}$
NUMBER OF RESONANCES	20	14	—
PUMP RATE λ_p	$\ll 2$ (3)	$\ll 8$ (3)	s^{-1}
FRACTION TRANSMUTED	—	$\ll 5$ (-5)	—

LSL

